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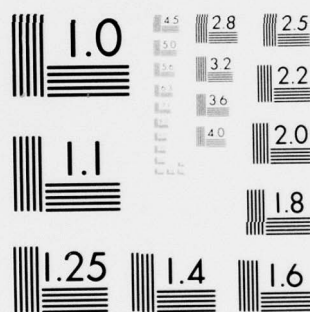
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Real Time Astrometry

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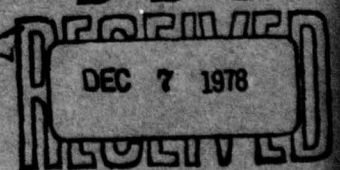
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This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

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REAL TIME ASTROMETRY

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### ABSTRACT

This report provides estimates of the positional accuracy for celestial objects which we have obtained at the Experimental Test Site of the Ground-based Electro-optical Deep Space Surveillance (GEODSS) program. Optical observations made at the ETS have been compared with radar measurements performed by the Millstone Hill Radar. A brief description of the equipment and theoretical basis of the optical data reduction is included too. It appears that optical observations of artificial satellites, with real time reduction, are accurate to  $\approx 5''$ .

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## I. INTRODUCTION

For twenty years the Baker-Nunn camera system has admirably served as our principal source of optical observations of artificial satellites. The United States Air Force and Lincoln Laboratory have recently completed the design, construction, and testing of the Experimental Test Site (ETS) of the Ground-based Electro-Optical Deep Space Surveillance (GEODSS) program. GEODSS is now currently being deployed world-wide by the TRW Company. When fully operational, it will supplant the Baker-Nunn photographic camera system. Moreover, because of various technological improvements, its limiting magnitude ( $\gtrsim 16^m$ ), search rate, and observing rate will far exceed those of the Baker-Nunn camera network. Hence, we can hope that some telescope time can be devoted to purely astronomical research (e.g., minor planet observations, supernovae searches, deep, multicolor, and very rapid catalogue construction to  $\approx 1''$ , variable star searches, meteor counts, etc.). For these reasons we believe that the positional accuracy of GEODSS's observations will be of interest to a large audience. Hence, this publication.

Below we briefly describe the hardware and computer facilities at the ETS. We then touch on some purely astrometrical matters and our handling of the SAOC (which serves as our reference star catalogue). Next we discuss the theoretical basis for the Precision Local Calibration (PLC) procedure currently employed by the ETS. This procedure rests directly on the reduction techniques of classical photographic astrometry. After presenting PLC we consider its expected accuracy and some simulation



testing we have performed. Observations made in May of 1978 on the brighter FK4 stars and eight artificial satellites have been used to experimentally verify the analytical and numerical work. Lastly, we describe a simpler, less accurate Single Star Calibration procedure which has been tested in the same way. Ultimately, we expect to use the plate overlap technique in real time (Taff<sup>1</sup>).

Briefly the results are as follows: The positional accuracy for the stars is  $1''.3 \pm 1''.0$  and  $\approx 5'' \pm 3''$  for the satellites. Secondly, relative to the FK4 itself for the stars or the Millstone Hill Radar for the artificial satellites, there is no systematic bias in right ascension or declination. Thirdly, there is no measurable color term, magnitude term, or systematic star-satellite term. In addition to these results, we wish to stress that the entire procedure (e.g., PLC or SSC) occurs in real time, under computer control, with an electronically generated finding chart, and requires approximately  $1^m.5$  ( $21^s$  for SSC) to complete. Furthermore, our theoretical computations and telescope alignment clearly indicate that we are only limited by the hardware from routinely splitting the seeing disc.

## II. EQUIPMENT

The telescope is a 31" Boller and Chivens f/5 of Ritchey-Chrétien design. At the Cassegrain focus the field of view is  $1^{\circ}16'$ . A zoom feature of the camera reduces this to  $34'.8$  (2:1). The low light level, high gain camera was built by Westinghouse following a design of R. Weber.<sup>2</sup> It consists of a Varian VLI-116 image intensifier coupled via fiber optics to a Westinghouse WX-32719 camera tube. The first photosurface is an 80 mm diameter S-20 type. The final target is a 32 mm diameter silicon diode array. Standard U.S.A. television frame rates and horizontal scan line densities were used for the work discussed herein. The camera target has 450 resels/line and the right ascension: declination: field of view ratio is 4:3:5. In the zoom configuration, the resolution element size is  $3'.7$ . The 50% response area has a diameter of  $56\mu$  on the camera target. The high ( $5 \times 10^4$ ) pre-readout gain ensures background noise limited performance with uncooled electronics.

The control of the telescope, the storage of the data files, and the data reduction are all handled by a MODCOMP IV computer. All the data files (star catalogues, Besselian day numbers, etc.) are stored using a 32 bit word for right ascension and declination ( $0''.10$  in each coordinate). All arithmetic is performed using a 48 bit word length.

The operator views the telescope display on a television monitor with a set of rectangular gridlines superimposed. While all gross movements of the telescope are under computer control, the final alignment of the star (or artificial satellite) relative to a fiducial mark in the grid system

is performed using a joystick type device. This control allows one to select one of  $2^8 = 256$  different horizontal and vertical settings and provides a 6.5 positioning uncertainty independent of the camera's resolution element.

A statistically independent combination of the telescope's shaft encoder's resolution (0.62), the atmospheric seeing disk ( $\approx 1''$ ), the camera's resolution cell, and the joystick's resolution cell imply that the standard deviation of a single measurement is  $\approx 5''$ . Experiments designed to test this result have indicated that this figure is probably closer to 4".

The observatory is located on the White Sands Missile Range near Socorro, New Mexico. Additional information about the observatory is contained in reference 2.

### III. POSITION REDUCTIONS

The source for positions and proper motions was the Smithsonian Astrophysical Observatory Star Catalog (Staff of the SAO, 1966). These 1950.0 values were updated for precession and proper motion to 1978.0 using the power series method (Newcomb<sup>3</sup>, Fricke and Kopff<sup>4</sup>, and Taff<sup>5</sup>). Foreshortening terms were not included. The accuracy is at least 0<sup>s</sup>.001 in right ascension and 0<sup>m</sup>.01 in declination for polar distances larger than 9<sup>o</sup> (exclusive of the foreshortening terms).

The reduction of the reference stars from catalogue mean place to topocentric mean place was performed using the Besselian day numbers A-E, J, and J' as well as including diurnal aberration. The astronomical refraction correction followed Smart<sup>6</sup>. The annual parallax correction was not included. A totally differential reduction from mean place to topocentric place has been implemented in which the reference stars are reduced relative to the center of the field. This allows one to recover the program object's mean place immediately after its topocentric place has been obtained from the telescope/camera model. It was these results that were directly compared (in the case of the stars) to 1978.0 values.

The reduction of the artificial satellites' position is performed identically. Both parallactic refraction and planetary aberration are small ( $\approx 2''$ ) and presently ignored. Complete details are in reference 5.



#### IV. THE TELESCOPE/CAMERA MODEL

Each field contains  $N$  reference stars and the program object. The topocentric position of a reference star is denoted by  $(\alpha, \delta)$  and the topocentric position of the program object is denoted by  $(A, \Delta)$ . The corresponding values as measured by the telescope are  $(\alpha_T, \delta_T)$  and  $(A_T, \Delta_T)$ . The model relates  $\{(\alpha, \delta)\}$  to  $\{(\alpha_T, \delta_T)\}$ . Then a prediction of  $(A, \Delta)$  is made from  $(A_T, \Delta_T)$  by inverting the model. It was assumed that a linear model would suffice\*. The equations below are analogous to the ones used when modeling a photographic plate in classical photographic astrometry. The intermediary device of standard coordinates was not used.

Let an angular bracket denote an equally weighted average over the  $N$  reference stars within a field. Furthermore, for each of the reference stars define

$$e_\alpha = \alpha - \alpha_T, \quad e_\delta = \delta - \delta_T, \quad (1a)$$

$$\text{and} \quad \epsilon_\alpha = \alpha - \langle \alpha \rangle, \quad \epsilon_\delta = \delta - \langle \delta \rangle. \quad (1b)$$

The corresponding quantities for the program object are,

$$E_\alpha = A - \langle \alpha \rangle \text{ and } E_\delta = \Delta - \langle \delta \rangle. \quad (2)$$

The full linear "plate model" is

$$e_\alpha = a\epsilon_\alpha + b\epsilon_\delta + c, \quad (3a)$$

$$e_\delta = d\epsilon_\alpha + e\epsilon_\delta + f, \quad (3b)$$

where  $a - f$  are the "plate constants".

---

\* Both the telescope and the television camera are being replaced so a detailed analysis of the geometry of the interface has been postponed.

The plate constants are determined by the procedures of least squares with equal weighting for each component of the model. Once this is done, the topocentric position of the program object is recovered by solving Eqs. (4);

$$A - A_T = aE_\alpha + bE_\delta + c, \quad (4a)$$

$$\Delta - \Delta_T = dE_\alpha + eE_\delta + f. \quad (4b)$$

The determinant of the normal equations is given by  $N\det(I)$  where

$$I = \begin{pmatrix} I_{\alpha\alpha} & I_{\alpha\delta} \\ I_{\delta\alpha} & I_{\delta\delta} \end{pmatrix} = I^T, \quad (5a)$$

and

$$I_{\alpha\alpha} = \sum \epsilon_\alpha \epsilon_\alpha, \quad I_{\alpha\delta} = \sum \epsilon_\alpha \epsilon_\delta, \quad I_{\delta\delta} = \sum \epsilon_\delta \epsilon_\delta. \quad (5b)$$

Hence,  $I$  is the "moment of inertia tensor" of the reference star distribution. Since  $\det(I)$  vanishes if and only if the reference stars lie on a straight line, as long as the problem is well posed, the solution for the plate constants exists and is unique. Furthermore, if  $\sigma_\alpha, \sigma_\delta$  denote the standard deviations of a single measurement in right ascension and declination, then from the covariance matrix of the plate constants,

$$\text{var}(A) = \{\sigma_\alpha^2 (1-e)^2 + \sigma_\delta^2 b^2\} \{1 + 1/N + \underline{E} \cdot I^{-1} \cdot \underline{E}\} / D^2, \quad (6a)$$

$$\text{var}(\Delta) = \{\sigma_\alpha^2 d^2 + \sigma_\delta^2 (1-a)^2\} \{1 + 1/N + \underline{E} \cdot I^{-1} \cdot \underline{E}\} / D^2, \quad (6b)$$

where

$$\underline{E} = (E_\alpha, E_\delta),$$

and  $D$  is the determinant of Eqs. (4),

$$D = (1-a)(1-e) - bd.$$

From Eqs. (6) we can conclude that the optimum position for the program object is at the center of the field (remember  $I$  is a positive definite quadratic form). Moreover, when the program object is so placed, the expected accuracy is independent of the areal extent of the reference stars and increasing the number of reference stars beyond the minimum value of 4 offers little improvement in accuracy (it's 2.3% for  $N=4 \rightarrow N=5$  and 11.8% for  $N=4 \rightarrow N=\infty$ ). While further rigorous analytical progress appears to be difficult, a heuristic averaging over the noise leads one to

$$\langle \text{var}(A) \rangle \approx \sigma_\alpha^2 [1 + 1/N + \underline{E} \cdot I^{-1} \cdot \underline{E}], \quad (7a)$$

$$\langle \text{var}(\Delta) \rangle \approx \sigma_\delta^2 [1 + 1/N + \underline{E} \cdot I^{-1} \cdot \underline{E}]. \quad (7b)$$

However,

$$\underline{E} \cdot I^{-1} \cdot \underline{E} = E_\alpha^2 / \text{var}(\epsilon_\alpha) + E_\delta^2 / \text{var}(\epsilon_\delta), \quad (8a)$$

so that if  $\underline{E} \neq \underline{0}$  one wants the maximum areal extent for the best accuracy.

In fact, if the right ascension and declination coordinates are treated symmetrically, then

$$\underline{E} \cdot I^{-1} \cdot \underline{E} \approx kE^2/(NS) \quad (8b)$$

where  $S$  is the area occupied by the reference stars and  $k$  is related to the radius of gyration of the distribution of reference stars about the

center of the field. See also Plummer<sup>7</sup>, Eichhorn and Williams<sup>8</sup>, and Taff<sup>9</sup>.

Extensive Monte Carlo simulations using a uniform distribution of reference stars and treating right ascension and declination symmetrically have confirmed Eqs. (7,8). Additional simulations were conducted using the SAOC itself. The field centers for the later computations were at  $\alpha = 0^h(0^h.5)23^h.5$ ,  $\delta = -85^\circ(5^\circ)85^\circ$ . While the SAOC simulations only used a circular field of  $0^\circ.75$  radius, the Monte Carlo trials used rectangular fields of edge size  $0^\circ.25$ ,  $0^\circ.50$ ,  $1^\circ.00$ ,  $2^\circ.00$ , and  $4^\circ.00$ . For both sets of computations  $\sigma_\alpha = \sigma_\delta = \sigma$  with  $\sigma = 1''.25$ ,  $2''.50$ ,  $5''.00$  or  $10''.00$  and the values of N used were 3(1)8. See reference 9 for complete details.



## V. RESULTS

### A. Stars

A total of thirty of the brighter FK4 stars were observed twice. The spacing in declination was about  $15^\circ$  (from  $-15^\circ$  to  $+60^\circ$ ). The spacing in hour angle was about  $2^h$  (from  $-4^h$  to  $+4^h$ ). Thus, the majority of the visible celestial hemisphere was covered.

The current version of PLC seeks a distribution of four stars uniformly spaced in position angle relative to the program object. If more than four stars are available, then the closest or brightest stars are preferentially chosen. In addition, the star closest to the program object is used as a control so that, normally, five stars are observed. Only four, however, are used in the modeling discussed in Section IV. The errors obtained for the control star (e.g., real position minus model position) are then added to the program object's model position. If there are less than four stars (including the control star) available within  $0.75^\circ$  of the program object's position, then the entire process is inhibited.

For the observations of the stars, the mean total positional error (e.g., mean  $\{[(\Delta\alpha\cos\delta)^2 + (\Delta\delta)^2]^{1/2}\}$ ) was  $1''.3$ . This had a standard deviation of  $1''.0$ . A separate series of measurements were made to test for a color or magnitude term. Over a range of  $1.2^m$  in B-V and  $4.5^m$  in V no significant term appeared.

## B. Artificial Satellites

The eight artificial satellites used to test the positional reduction accuracy of PLC are listed in Table I along with their orbital elements. For each of these satellites there are at least two independently determined sets of orbital elements available. This fact, their visibility, and the large fraction of the celestial sphere they cover prompted our choice of these satellites in particular.

The optical observations were interspaced with the radar observations to insure that station-keeping maneuvers would be detected. In addition, since the radar measurements both predated and postdated the optical ones, the continuity of the residuals could be assured. Precise estimates for the real accuracy of PLC (when using artificial satellites as test objects) are extremely difficult to obtain because of different models for the earth's surface, the geopotential's field, and small systematic errors in the measurement of position by the radar. Also, our neglect of planetary aberration is a  $\approx 2''$  systematic effect in right ascension for the small inclination satellites. The neglect of parallactic refraction is significant only for LAGEOS.

Standard software (TMPEST) was used to produce orbital element sets by using the radar data only and combining all of the data. We were able to achieve total positional errors as small as  $5''$  although the largest were  $15''$ . Further examination of the residuals showed that whenever the total positional error was larger than  $\approx 7.5''$  it was almost all in the declination. This was subsequently (and definitely) traced to a problem with the declination axis torque motor. The problem no longer exists.

TABLE 1  
SATELLITE ORBITAL ELEMENTS

NAME	n (rev/day)	e	i (deg)	$\Omega$ (deg)	$\omega$ (deg)	$M_0$ (deg)	$T_0$ (days)
GPS	2.01042	0.00285	63.1633	106.1344	101.3730	235.4767	191.0000
COMSTAR I	1.00279	0.00016	0.1284	73.8266	46.5219	39.6680	191.0000
COMSTAR II	1.00268	0.00016	0.1591	73.4403	13.9192	105.1594	191.0000
WESTAR II	1.00276	0.00005	0.0525	43.6527	43.0810	77.6249	191.0000
LAGEOS	6.38664	0.00441	109.8345	269.9748	93.1107	267.4310	177.6682
ATS-6	1.00279	0.00045	1.5485	83.9969	41.5144	22.2928	191.0000
IUE	1.00222	0.23929	28.5230	204.6788	260.2952	104.3550	191.0000
NIS-2	2.00577	0.00085	63.4025	103.6060	316.4364	1.3088	191.0000

## VI. SINGLE STAR CALIBRATION

Because telescope time is valuable, and for many purposes an artificial satellite's position need only be known to 10-15", we have developed a single star calibration procedure. This proceeds as PLC does except that the star in the SAOC closest to the input position is chosen to be the reference star. The program object's position is then differentially corrected for telescope errors and reduced (currently) to 1978.0. This latter reduction is done with an accuracy commensurate with that in the topocentric place itself. A non-technical overview of both PLC and SSC is contained in reference 10.



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